6.4 Factoring and Solving Polynomial Equations

Goals
- Factor polynomial expressions.
- Use factoring to solve polynomial equations.

Your Notes

VOCABULARY

Factor by grouping A method used to factor some polynomials with pairs of terms that have a common monomial factor. The pattern is $ra + rb + sa + sb = r(a + b) + s(a + b) = (r + s)(a + b)$.

Quadratic form The form $au^2 + bu + c$ where u is any expression in x

SPECIAL FACTORING PATTERNS

Sum of Two Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Example

$x^3 + 8 = (x + 2)(\underline{x^2 - 2x + 4})$

Difference of Two Cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example

$8x^3 - 1 = (2x - 1)(\underline{4x^2 + 2x + 1})$

Example 1 *Factoring the Sum or Difference of Cubes*

Factor each polynomial.

a. $x^3 - 64 = x^3 - \underline{4}^3$

$= (x - \underline{4})(\underline{x^2 + 4x + 16})$

b. $54y^4 + 16y = 2y(\underline{27y^3 + 8})$

$= 2y\left[(\underline{3y})^3 + \underline{2}^3\right]$

$= 2y(\underline{3y + 2})(\underline{9y^2 - 6y + 4})$

Your Notes

Example 2 *Factoring by Grouping*

Factor the polynomial $x^3 - 3x^2 - 4x + 12$.

Solution

$x^3 - 3x^2 - 4x + 12$

$= x^2(\underline{x - 3}) - 4(\underline{x - 3})$ **Factor by grouping.**

$= \underline{(x^2 - 4)(x - 3)}$

$= \underline{(x - 2)(x + 2)(x - 3)}$ **Difference of squares**

Example 3 *Factoring Polynomials in Quadratic Form*

Factor (a) $16x^4 - 1$ and (b) $2x^6 - 10x^4 + 12x^2$.

Solution

a. $16x^4 - 1 = (\underline{4x^2})^2 - \underline{1}^2$

$= \underline{(4x^2 + 1)(4x^2 - 1)}$

$= \underline{(4x^2 + 1)(2x - 1)(2x + 1)}$

b. $2x^6 - 10x^4 + 12x^2 = 2x^2(\underline{x^4 - 5x^2 + 6})$

$= \underline{2x^2(x^2 - 3)(x^2 - 2)}$

Example 4 *Solving a Polynomial Equation*

Solve $x^4 + 4 = 5x^2$.

Solution

$x^4 + 4 = 5x^2$ **Write original equation.**

$\underline{x^4 - 5x^2 + 4} = 0$ **Rewrite in standard form.**

$\underline{(x^2 - 4)(x^2 - 1)} = 0$ **Factor trinomial.**

$\underline{(x - 2)(x + 2)(x - 1)(x + 1)} = 0$ **Factor difference of squares.**

$x = \underline{2}$, $x = \underline{-2}$, $x = \underline{1}$, or $x = \underline{-1}$ **Zero product property**

The solutions are $\underline{-1, 1, -2, \text{ and } 2}$. Check these in the original equation.

Your Notes

Checkpoint Factor each polynomial in Exercises 1–3.

1. $x^3 + 216$

$(x + 6)(x^2 - 6x + 36)$

2. $x^3 - x^2 - 2x + 2$

$(x^2 - 2)(x - 1)$

3. $x^4 - 7x^2 + 12$

$(x^2 - 3)(x + 2)(x - 2)$

4. Solve $x^5 - 2x = -x^3$.

$0, -1, 1$

Example 5 *Solving a Polynomial Equation in Real Life*

A rectangular swimming pool has a volume of 512 cubic feet. The pool's dimensions are x feet deep by $6x - 8$ feet long by $6x - 16$ feet wide. How deep is the pool?

Verbal Model $\boxed{\text{Volume}} = \boxed{\text{Depth}} \cdot \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Labels

Volume = 512 (cubic feet)

Depth = x (feet)

Length = $6x - 8$ (feet)

Width = $6x - 16$ (feet)

Algebraic Model

$512 = x(6x - 8)(6x - 16)$

$0 = 36x^3 - 144x^2 + 128x - 512$ **Standard form**

$0 = 36x^2(x - 4) + 128(x - 4)$ **Factor by grouping.**

$0 = (36x^2 + 128)(x - 4)$

The only real solution is $x = $ 4, so $6x - 8 = $ 16 and $6x - 16 = $ 8. The pool is 4 feet deep. The dimensions are 4 feet by 16 feet by 8 feet.

Homework